CENTRAL UNIVERSITY OF HARYANA

Even Semester Term End Examinations June 2023

Programme: Integrated B.Sc.-M. Sc. (Mathematics) | **Session: 2022-23**

Semester: Fourth | **Max. Time: 3 Hours**

Course Title: Linear Algebra | **Max. Marks: 70**

Course Code: SBSMAT 03 04 02 C 5106

Instructions:

1. Question no. 1 has seven parts and students are required to answer any five. Each part carries two marks.

2. Questions no. 2 to 6 have three questions and students are required to answer any two parts of each question. Each part carries six marks.

Q 1. (5X2=10)

a) Do you think that the union of two subspaces is a subspace or not? Give an example in support of your answer.

b) What do you mean by isomorphism? Give an example of two vector spaces which are isomorphic.

c) Show whether the following is a linear transformation or not: $T: \boldsymbol{R}^3 \to \boldsymbol{R}^2$ defined by $T(x, y, z) = (|x|, 0)$.

d) State the Cayley-Hamilton theorem.

e) Let $\emptyset$ be a linear functional on $\boldsymbol{R}^2$ defined by $\emptyset(x, y) = x - 2y$. Let T be a linear operator on $\boldsymbol{R}^2$ defined by $T(x, y) = (y, x + y)$. Find $(T^t(\emptyset))(x, y)$.

f) Define orthogonality in an inner product space.

g) Define Unitary and Normal linear operator.

Q 2. (2X5=10)

a) Show that the set $\boldsymbol{Q}(\sqrt{2}) = \{a + b\sqrt{2}: a, b \in \boldsymbol{Q}\}$ is a vector space over **Q** with respect to the compositions: $(a + b\sqrt{2}) + (c + d\sqrt{2}) = (a + c + (b + d)\sqrt{2})$, $\alpha(a + b\sqrt{2}) = \alpha a + \alpha b\sqrt{2}$ where a, b, c, d and α are all rational numbers.

b) If $v_1 = (1, 2, -1), v_2 = (2, -3, 2), v_3 = (4, 1, 3), v_4 = (-3, 1, 2) \in \boldsymbol{R}^3(\boldsymbol{R})$. Prove that $L(\{v_1, v_2\}) \neq L(\{v_3, v_4\})$.

c) State and prove the Extension theorem for the basis of a finite-dimensional vector space.

Q3. (2X5=10)

a) Let $T_1: \boldsymbol{R}^3 \to \boldsymbol{R}^2$ *and* $T_2: \boldsymbol{R}^3 \to \boldsymbol{R}^2$ are two linear transformations defined by $T_1(x, y, z) = (3x, y + z)$ *and* $T_2(x, y, z) = (2x - z, y)$. Find (i) $T_1 + T_2$ (ii) $3T_1 - 4T_2$ (iii) $T_1 o T_2$ (iv) $T_2 o T_1$

b) A linear transformation $T: U \to V$ is one-to-one if and only if $T(u) = 0 \Rightarrow u = 0$.

c) State and prove Rank-Nullity Theorem.

Q 4. (2X5=10)

a) Find the dual basis of the basis $\{(-1,1,1),(1,-1,1),(1,1,-1)\}$ of $\boldsymbol{R}^3(\boldsymbol{R})$.

b) Find the eigenvalues, eigenvectors and eigenspace of the following linear operator $T: \boldsymbol{R}^3 \to \boldsymbol{R}^3$ defined by $T(x,y,z) = (x+6y-2z, -3x+2y, 3y-4z)$.

c) Let T be a linear operator on $\boldsymbol{R}^2(\boldsymbol{R})$ defined by $T(x,y) = (x+y, x-y)$. Find the characteristic polynomial and minimal polynomial for T.

Q 5. (2X5=10)

a) For what values of k is the following an inner product space on $\boldsymbol{R}^2$ $\langle u,v\rangle = x_1y_1 - 3x_1y_2 - 3x_2y_1 + kx_2y_2$ where $u = (x_1,x_2)$ and $v = (y_1,y_2) \in \boldsymbol{R}^2$.

b) State and prove Cauchy-Schwarz's inequality.

c) Consider the subspace U of $\boldsymbol{R}^4$ spanned by the vectors $v_1 = (1,1,1,1), v_2 = (1,1,2,4)$, $v_3 = (1,2,-4,-3)$. Find (a) an orthogonal basis of U
(b) an orthonormal basis of U.

Q 6. (2X5=10)

a) Show that any operator T on an inner product space V is the sum of a self-adjoint operator and a skew-adjoint operator.

b) Find all possible Jordan canonical forms for a linear map $T: V \to V$ whose characteristic polynomial is $(t-7)^5$ and whose minimal polynomial is $(t-7)^2$.

c) What do you mean by invariant subspace? Let $T: V \to V$ be linear and let $f(t)$ be any polynomial. Show that kernel of $f(T)$ is invariant under T.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations June-2023

Programme	: M.Sc. Mathematics	Session	: 2022-23
Semester	: Fourth	Max. Time	: 3 Hours
Course Title	: Measure Theory and Integration	Max. Marks	: 70
Course Code	: SBSMAT 01 04 08 DCEC 3104		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Show that outer measure of $\mathbb{Q}$ (set of rational numbers) is zero.
   (b) If $m^*(A) = 0$ then prove that $m^*(A \cup B) = m^*(B)$.
   (c) If a set E is measurable, then so is its complement E^c.
   (d) Show that a countable union of null sets in a measure space is a null set of the measure space.
   (e) If f and g are measurable functions on a common domain E, then the set
   $$A = \{x \in E : f(x) < g(x)\}$$
   is measurable.
   (f) If f is measurable function then $|f|$ is also measurable function and prove converse of this is not true.
   (g) Let μ be counting measure on $\mathbb{N}$. Interpret Fatou's lemma, monotone and dominated convergence theorems as statements about infinite series.

2. (a) Let $\{A_n\}$ be a countable collection of sets of real numbers. Then $m^*(\cup A_n) \leq \sum m^*(A_n)$.
   (b) The collection M of measurable sets is a σ- algebra.
   (c) Let $\{E_n\}$ be an infinite sequence of measurable sets such that $E_{n+1} \subseteq E_n$ for each n. Then $m(\cap_{n=1}^{\infty} E_n) = \lim_{n\to\infty} m(E_n)$.

3. (a) Let f be a function defined on $[0, \frac{1}{\pi}]$ as follows $f(x) = \begin{cases} 0.1 & \text{if } x = 0 \\ 2x sin\frac{1}{x} & \text{if } x > 0 \end{cases}$ determine the measure of the set $\{x : f(x) \geq 0\}$.
   (b) Give an example of an uncountable set of measure zero.
   (c) Prove that there exists a non-measurable set in interval [0, 1).

4. (a) Prove that every continuous function is measurable and give an example of a measurable discontinuous function.
   (b) State and prove Lusin's Theorem.
   (c) If a sequence $< f_n >$ converges in measure to f on a measurable set E, then there exist a sub-sequence $< f_{n_k} >$ which converges to f on E.

5. (a) State and prove Fatou's lemma.

(b) By using Lebesgue dominated convergence theorem find the Lebesgue integral of $\int_0^1 \frac{x\ sinx}{1+(nx)^\alpha}dx$ where $\alpha > 1$.

(c) State and prove bounded convergence theorem.

CENTRAL UNIVERSITY OF HARYANA
Fourth Semester Examinations June-2023

Programme	: Integrated B.Sc.-M.Sc.(Mathematics)	Session	: 2023-2024
Semester	: IV	Max. Time	: 3 Hours
Course Title	: Partial Differential Equations and Calculus of Variations	Maximum Marks	: 70
Course Code	: SBSMAT 03 04 03 C 5106		

Instructions:

1. Question no. 1 has has seven sub parts and students need to answer any five. Each sub part carries two Marks.
2. Question no. 2 to 6 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries six marks.

1. (a) Define Partial differential equation(PDE). Give one example of linear PDE.

   (b) Define degree of PDE. Give one example of 2nd degree PDE.

   (c) Form the PDE by eliminating arbitrary constants from the relation $Z = (x+a)(y+b)$.

   (d) Define Non-Homogeneous PDE of order n. Give one example.

   (e) Classify the PDE $\frac{\partial^2 u}{\partial x^2} - \frac{\partial z}{\partial y} = 0$.

   (f) Show that the variation of a functional is zero on curves on which extremum of a functional is achieved.

   (g) Define variational problems and calculus of variation.

2. (a) Write a short note on formation of PDE. Find the PDE of the family of spheres of radius 7 with centres on the plane $x - y = 0$.

   (b) Explain Lagrange's method for type four problems. Solve by separation of variables

   $$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} - 2u \text{ with } u(x,0) = 10e^{-x} - 6e^{-4x}.$$

   (c) Explain the working rules for Charpit's general method. Solve PDE px+qy=pq by Charpit's method obtain the general solution and show that there is no singular solution.

3. (a) Give complete description of method to find the complementary function of linear Homogeneous PDE with constant coefficients $F(D, D')z = f(x,y)$.

   (b) Find the general solution of the following PDE

   $$(D^3 + D^2D' - DD'^2 - D'^3)z = e^x cos2y.$$

   (c) Solve $(D^2 + DD' - 6D'^2)z = x^2 sin(x+y)$.

4. (a) Find whether the PDE

   $$x\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial y \partial x} + y\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} = 0$$

   is hyperbolic, parabolic or elliptic in nature.

   (b) Classify and reduce the equation $\frac{\partial^2 z}{\partial x^2} - x^2\frac{\partial^2 z}{\partial y^2} = 0$ to canonical form.

(c) Solve $r = a^2 t$ by Monge's method.

5. (a) Define linear functional. Show that the functional

$$I[y(x)] = \int_a^b \{y'(x) + y(x)\}dx$$

is linear in the class $C'[a, b]$.

(b) State and prove the fundamental lemma of the calculus of variations.

(c) State Euler's equation. Show that the Euler's equation for the functional

$$I[y(x)] = \int_{x_1}^{x_2} \{a(x)y'^2 + 2b(x)yy' + c(x)y^2\}dx$$

is a second order linear differential equation.

6. (a) Show that the necessary conditions for the extremum of the functional

$$I[y(x)] = \int_{x_1}^{x_2} F(x, y, y')dx$$

in the case when the variation δx_2 and δy_2 are independent, are given by $[F - y'Fy']_{x=x_2} = 0$ and $[Fy']_{x=x_2} = 0$.

(b) Find the shortest distance between point $P_1(1, 0)$ and the ellipse $4x^2 + 9y^2 = 36$.

(c) Find the shortest distance of the point $(0, 2, 4)$ to the straight line $\frac{x-1}{1} = \frac{y}{3} = \frac{z}{4}$.

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, June-2023

Programme	: M.Sc. Mathematics	Session	: 2023-24
Semester	: Fourth	Max. Time	: 3 Hours
Course Title	: Differential Geometry	Maximum Marks	: 70
Course Code	: SBSMAT 01 04 01 DCEC 3104		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each part carries three and half marks.
2. Question no. 2 to 5 have three parts and students need to answer any two parts of each question. Each part carries seven marks.

1. (a) Define curvature and torsion.
   (b) Find the equation of the plane that has three point contact at the origin with the curve $x = t^4 - 1, y = t^3 - 1, z = t^2 - 1$.
   (c) Explain Envelope and characteristic curve of family of surfaces.
   (d) What do you mean by principal directions and curvatures?
   (e) Derive the relation between the coefficients E, F, G and H.
   (f) Discuss the nature of geodesics on a sphere.
   (g) Define geodesic ellipses and hyperbolas.

2. (a) Show that curvature and torsion are both constant for the circular helix $\mathbf{r} = (a\cos\theta, a\sin\theta, a\theta\cot\alpha)$.
   (b) Find the curvature and torsion of the locus of the centre of spherical curvature.
   (c) Derive the Serret-Frenet formulae.

3. (a) State and prove Euler's theorem.
   (b) Show that a necessary and sufficient condition for a surface to be developable surface is that its Gaussian curvature shall be zero.
   (c) Prove that the edge of regression of the polar developable of a space curve is the locus of the centre of spherical curvature.

4. (a) Derive the Rodrigue's formula.
   (b) Show that the principal radii of curvature of the surface $y\cos\left(\frac{z}{a}\right) = x\sin\left(\frac{z}{a}\right)$ are equal to $\pm\frac{(x^2+y^2+z^2)}{a}$. Find the lines of curvature.
   (c) Prove that the mean curvature is zero at every point on the surface of revolution $\mathbf{r} = (\cosh u\cos v, \cosh u\sin v, u)$.

5. (a) State and prove Joachimsthal's theorems.
   (b) Derive differential equation of geodesics by using normal property.
   (c) Obtain Liouville's formula for geodesic curvature of a curve.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations June 2023

Programme	: Integrated B.Sc.-M.Sc., B.Tech.	Session	: 2022-2023
Semester	: Fourth	Max. Time	: 3 Hours
Course Title	: Vector Calculus	Max. Marks	: 70
Course Code	: SBSMAT 03 04 01 GE 5106		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any five. Each sub part carries two marks.
2. Question no. 2 to 6 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries six marks.

1. (a) If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{c} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$, find the value of $[(\vec{a} \times \vec{b}) \times \vec{c}]$.
   (b) If $\vec{r} = (t - t^2)\hat{\imath} + 2t^3\hat{\jmath} - 3\hat{k}$, find $\int_2^3 \vec{r}\, dt$.
   (c) If $\Phi = x^2yz$, $\Psi = xy - 3z^2$, then find $\nabla(\nabla\Phi.\nabla\Psi)$.
   (d) Find λ, μ and ν so that the vector
   $$\vec{f} = (2x + 3y + \lambda z)\hat{\imath} + (\mu x + 2y + 3z)\hat{\jmath} + (2x + \nu y + 3z)\hat{k}$$
   is irrotational.
   (e) Show that $\oint_C \Phi\nabla\Phi.\vec{dr} = 0$, C being a closed curve.
   (f) Prove that $\hat{e_1} = h_2h_3\, \nabla v \times \nabla w$, with similar results for $\hat{e_2}$ and $\hat{e_3}$ where u, v, w are orthogonal co-ordinates.
   (g) Define scalar, vector and tensor by giving example of each.

2. (a) Show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $\vec{a} - 3\vec{b} + 5\vec{c}$ are coplanar.
   (b) If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$, $\vec{c} = \hat{\imath} - \hat{\jmath} + \hat{k}$, find $\vec{a} \times (\vec{b} \times \vec{c})$ and verify that $\vec{a} \times (\vec{b} \times \vec{c})$ is perpendicular to both $\vec{a}$ and $\vec{b} \times \vec{c}$.
   (c) The acceleration of a particle at any time t is given as $e^t\hat{\imath} + e^{2t}\hat{\jmath} + \hat{k}$. Find $\vec{v}$, given that $\vec{v} = \hat{\imath} + \hat{\jmath}$ when $t = 0$.

3. (a) Evaluate $\iint_S \vec{f}.\hat{n}\, dS$, where $\vec{f} = (x + y^2)\hat{\imath} - 2x\hat{\jmath} + 2yz\hat{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.
   (b) If $\vec{f} = (2x^2 - 3z)\hat{\imath} - 2xy\hat{\jmath} - 4x\hat{k}$, evaluate $\iiint_V \nabla \times \vec{f}\, dV$, where V is the region bounded by the coordinate planes and the plane $2x + 2y + z = 4$.
   (c) Find the circulation of $\vec{f}$ around the curve C, where $\vec{f} = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ and C is the circle $x^2 + y^2 = 1$, $z = 0$.

4. (a) State and prove Gauss's Divergence Theorem.
   (b) Verify Stoke's theorem for the function $\vec{f} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$ taken round the rectangle bounded by $x = \pm a$, $y = 0$, $y = b$.
   (c) Evaluate $\iint_S \vec{f}.\hat{n}\, dS$, where $\vec{f} = 4xy\hat{\imath} + yz\hat{\jmath} - xz\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$. $z = 0$, $z = 2$.

5. (a) Prove that cylindrical co-ordinate system is orthogonal.
   (b) Express $x\hat{i} + 2y\hat{j} + yz\hat{k}$ in spherical co-ordinates.
   (c) Show that $grad\ \phi = \frac{\hat{e_1}}{h_1}\frac{\partial \phi}{\partial u} + \frac{\hat{e_2}}{h_2}\frac{\partial \phi}{\partial v} + \frac{\hat{e_3}}{h_3}\frac{\partial \phi}{\partial w}$ in curvilinear co-ordinates.

6. (a) Describe Rank of tensor and all of its type with examples.
   (b) Show that $\frac{\partial \Phi}{\partial x^i}$ is a covariant vector where Φ is a scalar function.
   (c) Define Kronecker delta and show that Kronecker delta is a mixed tensor of rank two.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations June 2023

Programme	: Integrated B.Sc.-M.Sc., B.Tech.	Session	: 2022-2023
Semester	: Fourth	Max. Time	: 3 Hours
Course Title	: Vector Calculus	Max. Marks	: 70
Course Code	: SBSMAT 03 04 01 GE 5106		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any five. Each sub part carries two marks.
2. Question no. 2 to 6 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries six marks.

1. (a) If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + \hat{j} + 2\hat{k}$, find the value of $[(\vec{a} \times \vec{b}) \times \vec{c}]$.

   (b) If $\vec{r} = (t - t^2)\hat{i} + 2t^3\hat{j} - 3\hat{k}$, find $\int_2^3 \vec{r}\, dt$.

   (c) If $\Phi = x^2yz$, $\Psi = xy - 3z^2$, then find $\nabla(\nabla\Phi . \nabla\Psi)$.

   (d) Find λ, μ and ν so that the vector

$$\vec{f} = (2x + 3y + \lambda z)\hat{i} + (\mu x + 2y + 3z)\hat{j} + (2x + \nu y + 3z)\hat{k}$$

   is irrotational.

   (e) Show that $\oint_C \Phi \nabla\Phi . \vec{dr} = 0$, C being a closed curve.

   (f) Prove that $\hat{e_1} = h_2 h_3 \nabla v \times \nabla w$, with similar results for $\hat{e_2}$ and $\hat{e_3}$ where u, v, w are orthogonal co-ordinates.

   (g) Define scalar, vector and tensor by giving example of each.

2. (a) Show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $\vec{a} - 3\vec{b} + 5\vec{c}$ are coplanar.

   (b) If $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, $\vec{c} = \hat{i} - \hat{j} + \hat{k}$, find $\vec{a} \times (\vec{b} \times \vec{c})$ and verify that $\vec{a} \times (\vec{b} \times \vec{c})$ is perpendicular to both $\vec{a}$ and $\vec{b} \times \vec{c}$.

   (c) The acceleration of a particle at any time t is given as $e^t\hat{i} + e^{2t}\hat{j} + \hat{k}$. Find $\vec{v}$, given that $\vec{v} = \hat{i} + \hat{j}$ when $t = 0$.

3. (a) Evaluate $\iint_S \vec{f}.\hat{n}\, dS$, where $\vec{f} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

   (b) If $\vec{f} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, evaluate $\iiint_V \nabla \times \vec{f}\, dV$, where V is the region bounded by the coordinate planes and the plane $2x + 2y + z = 4$.

   (c) Find the circulation of $\vec{f}$ around the curve C, where $\vec{f} = y\hat{i} + z\hat{j} + x\hat{k}$ and C is the circle $x^2 + y^2 = 1$, $z = 0$.

4. (a) State and prove Gauss's Divergence Theorem.

   (b) Verify Stoke's theorem for the function $\vec{f} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by $x = \pm a$, $y = 0$, $y = b$.

   (c) Evaluate $\iint_S \vec{f}.\hat{n}\, dS$, where $\vec{f} = 4xy\hat{i} + yz\hat{j} - xz\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$, $z = 2$.

5. (a) Prove that cylindrical co-ordinate system is orthogonal.

   (b) Express $x\hat{i} + 2y\hat{j} + yz\hat{k}$ in spherical co-ordinates.

   (c) Show that $grad\ \phi = \frac{\hat{e_1}}{h_1}\frac{\partial\phi}{\partial u} + \frac{\hat{e_2}}{h_2}\frac{\partial\phi}{\partial v} + \frac{\hat{e_3}}{h_3}\frac{\partial\phi}{\partial w}$ in curvilinear co-ordinates.

6. (a) Describe Rank of tensor and all of its type with examples.

   (b) Show that $\frac{\partial\Phi}{\partial x^i}$ is a covariant vector where Φ is a scalar function.

   (c) Define Kronecker delta and show that Kronecker delta is a mixed tensor of rank two.

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, June-2023

Programme	: Integrated B.Sc.-M.Sc. Mathematics (Re-appear)	Session	: 2022-23
Semester	: Second	Max. Time	: 3 Hours
Course Title	: Real Analysis	Max. Marks	: 70
Course Code	: SBSMAT 03 02 01 C 5106/ SBSMAT 03 02 03 GE 5106		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Define countable and uncountable set with example. $(4 \times 3.5 = 14)$
   (b) Prove that intersection of two open sets is open set.
   (c) Define oscillatory sequences with example.
   (d) State Bolzano-Weierstrass theorem for sets.
   (e) State Cauchy second theorem on limit.
   (f) State Cauchy convergence criterion for an infinite series.
   (g) Explain D'Alembert's ratio test.

2. (a) Prove that the set of rational numbers is not an order complete field. $(7 \times 2 = 14)$
   (b) Prove that the unit interval $[0, 1]$ is uncountable.
   (c) State and prove Archimedian property of real numbers.

3. (a) Prove that a finite set has no limit point. $(7 \times 2 = 14)$
   (b) Show that every convergent sequence has a unique limit point.
   (c) Prove that every bounded and infinite set has a limit point.

4. (a) State and prove Cauchy first theorem on limits. $(7 \times 2 = 14)$
   (b) Prove that the sequence $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$ is convergent and its limit lies between $\frac{1}{2}$ and 1.
   (c) Prove that every convergent sequence is bounded.

5. (a) Discuss the convergence of the series $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^n, \quad x > 0.$ $(7 \times 2 = 14)$
   (b) Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$.
   (c) Prove that every absolutely convergent series is convergent. Test the absolute convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations June 2023

Programme	: Integrated B.Sc.-M.Sc., B.Tech.	Session	: 2022-2023
Semester	: Fourth	Max. Time	: 3 Hours
Course Title	: Vector Calculus	Max. Marks	: 70
Course Code	: SBSMAT 03 04 01 GE 5106		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any five. Each sub part carries two marks.
2. Question no. 2 to 6 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries six marks.

1. (a) If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + \hat{j} + 2\hat{k}$, find the value of $[(\vec{a} \times \vec{b}) \times \vec{c}]$.

   (b) If $\vec{r} = (t - t^2)\hat{i} + 2t^3\hat{j} - 3\hat{k}$, find $\int_2^3 \vec{r}\, dt$.

   (c) If $\Phi = x^2yz$, $\Psi = xy - 3z^2$, then find $\nabla(\nabla\Phi.\nabla\Psi)$.

   (d) Find λ, μ and ν so that the vector
   $$\vec{f} = (2x + 3y + \lambda z)\hat{i} + (\mu x + 2y + 3z)\hat{j} + (2x + \nu y + 3z)\hat{k}$$
   is irrotational.

   (e) Show that $\oint_C \Phi\nabla\Phi.\vec{dr} = 0$, C being a closed curve.

   (f) Prove that $\hat{e_1} = h_2h_3\, \nabla v \times \nabla w$, with similar results for $\hat{e_2}$ and $\hat{e_3}$ where u, v, w are orthogonal co-ordinates.

   (g) Define scalar, vector and tensor by giving example of each.

2. (a) Show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $\vec{a} - 3\vec{b} + 5\vec{c}$ are coplanar.

   (b) If $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, $\vec{c} = \hat{i} - \hat{j} + \hat{k}$, find $\vec{a} \times (\vec{b} \times \vec{c})$ and verify that $\vec{a} \times (\vec{b} \times \vec{c})$ is perpendicular to both $\vec{a}$ and $\vec{b} \times \vec{c}$.

   (c) The acceleration of a particle at any time t is given as $e^t\hat{i} + e^{2t}\hat{j} + \hat{k}$. Find $\vec{v}$, given that $\vec{v} = \hat{i} + \hat{j}$ when $t = 0$.

3. (a) Evaluate $\iint_S \vec{f}.\hat{n}\, dS$, where $\vec{f} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

   (b) If $\vec{f} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, evaluate $\iiint_V \nabla \times \vec{f}\, dV$, where V is the region bounded by the coordinate planes and the plane $2x + 2y + z = 4$.

   (c) Find the circulation of $\vec{f}$ around the curve C, where $\vec{f} = y\hat{i} + z\hat{j} + x\hat{k}$ and C is the circle $x^2 + y^2 = 1$, $z = 0$.

4. (a) State and prove Gauss's Divergence Theorem.

   (b) Verify Stoke's theorem for the function $\vec{f} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by $x = \pm a$, $y = 0$, $y = b$.

   (c) Evaluate $\iint_S \vec{f}.\hat{n}\, dS$, where $\vec{f} = 4xy\hat{i} + yz\hat{j} - xz\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$, $z = 2$.

5. (a) Prove that cylindrical co-ordinate system is orthogonal.
   (b) Express $x\hat{i} + 2y\hat{j} + yz\hat{k}$ in spherical co-ordinates.
   (c) Show that $grad\ \phi = \frac{\hat{e_1}}{h_1}\frac{\partial \phi}{\partial u} + \frac{\hat{e_2}}{h_2}\frac{\partial \phi}{\partial v} + \frac{\hat{e_3}}{h_3}\frac{\partial \phi}{\partial w}$ in curvilinear co-ordinates.

6. (a) Describe Rank of tensor and all of its type with examples.
   (b) Show that $\frac{\partial \Phi}{\partial x^i}$ is a covariant vector where Φ is a scalar function.
   (c) Define Kronecker delta and show that Kronecker delta is a mixed tensor of rank two.

CENTRAL UNIVERSITY OF HARYANA

3rd Semester Term End Examinations June 2023

Program: M.Sc Mathematics **Session:2022-23**

Course Title: Integral Equations and Calculus of Variation

Course Code: SBSMAT 01 03 01 C 3104

Max. Time: 3 Hours **Max. Marks: 70**

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Questions no. 2 to 5 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Define the eigenvalues and eigenfunctions of a kernel.

b) Classify the following integral equations $v(x)y(x) = e^x + \lambda \int_0^1 \text{xt}\, y(t)\text{dt},\ v \neq 0.$

c) For the integral equation $y(x) = 1 + x^3 + \int_0^x 2^{x-t}\, y(t)dt$, find the iterated kernel $K_3(x,t)$.

d) Solve the following integral equation $y(x) = tanx + 1\int_{-1}^{1} e^{cos^{-1}x} y(t)dt.$

e) Prove that the Euler's equation is invariant.

f) Discuss the broken extremals.

g) If $f(s)$is the Fourier transform of $F(x)$then find the Fourier transform of $F(-\text{bx})$.

Q 2. (2X7=14)

a) Prove that the solutions of the integral equation $y(x) = \frac{1}{2}x^2 + \int_0^x t(t-x)\, y(t)dt$

is the solution of the following initial value problem $y'' + xy = 1, y(0) = y'(0) = 0$

b) Solve the following integral equation by successive approximation method

$y(x) = 1 - xsinx + xcosx + \int_0^x ty(t)dt,$ and then obtain the solution in closed form.

c) Solve the following Volterra integral equations by reducing into the Volterra integral equations of the second kind

$$x^2 = \int_0^x (2 + x^2 - t^2)\, y(t)dt.$$

Q3. (2X7=14)

a) Reduce the following boundary value problem into the integral equation and classify the obtained integral equation

$$y''(t) + g(t)y(t) = h(t), \quad 0 < t < 1,\ y(0) = \alpha.\ \ y(1) = \beta.$$

b) Determine the eigenvalues and eigenfunctions of the following integral equation

$$y(x) = \lambda \int_0^{2\pi} sin^3t\, y(t)dt.$$

c) Derive the Neumann Series for Fredholm integral equation.

Q 4. (2X7=14)

(a) (i) Find the extremals of the functional

$$J[y]=\int_0^{\pi/2}(y^2-y'^2-2y\sin x)dx,\ y(0)=1, y(\pi/2)=1$$

(ii) Find the plane curve of fixed perimeter and maximum area.

(b) Find the shortest distance between the curves $y=9x^2$ and $x-y=2$.

(c) Discuss the Jacobi's and Legendre conditions for the extremum of the functional

$$J[y]=\int_0^1\left(\frac{x^2y'^2}{2}-2xyy'+y\right)dx, u(0)=0,\ u=\delta y$$ further, find the extremal satisfy and emanating from $\left(2,\frac{3}{2}\right)$.

Q 5. (2X7=14)

a) (i) Find the $L^{-1}\left[\frac{s+1}{s^2(s+1)^3}\right]$.

(ii) Find $L\left[\frac{cosat-cosbt}{t}\right]$.

b) (i) Using Laplace transform, solve $\int_0^x\frac{y(t)}{(x-t)^{1/2}}dt=1+x+x^2$.

(ii) Using Laplace transform, solve $y'(x)=x+\int_0^x y(x-t)cost\,dt,\ y(0)=4$.

c) Find the Fourier sine and Fourier cosine transforms of $(x)=\begin{cases}1+x, & 0<x<1\\ 2+x, & 1<x<2\\ 0, & x>2\end{cases}$.

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, June/july-2023

Programme	: M.Sc. Mathematics		
Semester	: Second	Session	: 2022-2023
Course Title	: Numerical Methods	Max. Time	: 3 Hours
Course Code	: SBSMAT 01 02 02 GEC 2124	Maximum Marks	: 70

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. $(4 \times 3.5 = 14)$

(a) Derive the Order of Convergence of Newton Raphson Method.

(b) Given the IVP $\frac{dy}{dx} = x - y^2$,$y(1) = 2$ to obtain the value of $y(1.5)$ with the help of Euler method with step size $h = 0.1$

(c) Use Trapezoidal rule to evaluate the integral $\int_0^1 sin(x)dx$ correct to two decimal places with step size $h = 0.5$

(d) Compute the value of $f(1.35)$ by using Bessel central difference formula :

x:	1	1.25	1.5
f(x):	0	0.223144	0.405465

with step size $h = 0.25$

(e) Approximate the integral $\int_2^4 \frac{e^x - cos(x)}{x^2}dx$ with the help of Gauss-Chebyshev 3-points formula .

(f) Construct the Lagrange interpolating polynomial for the following set :

x:	0	0.1	0.2	0.4
f(x):	0.74	1.2	1.87	5.15

(g) Use intermediate value theorem to prove that the equation $x^3 - 4x - 9 = 0$ has a root in the interval $(2, 3)$ .Obtain five iteration of the Bisection method to compute the approximate root of the equation.

2. (a) Find the positive root of the equation $xe^x = 2$, which lies in the interval $(0, 1)$ and correct upto four decimal places by using Secant method. [7]

(b) Solve the following system of linear equation with the help of Gauss Jordan method

$$2x_1 - x_2 + 3x_3 = 6$$

$$x_1 + 3x_2 - x_3 = -3$$

$$-2x_1 - x_2 - 3x_3 = -4$$

with initial approximation $(0, 0, 0)$ [7]

(c) Solve the following system of linear equation with the help of Gauss Seidel method with the initial appproximation $(0, 0, 0)$

$$2x + 6y - z = 9$$

$$2x + y + 14z = -11$$

$$6x + 4y + z = 9$$

[7]

3. (a) Determine the largest eigenvalue and the corresponding eigenvector of the matrix :

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

to 3 correct decimal places using the Power method. [7]

(b) Apply Stirling formulas to find the value of $f(37.5)$ from the following table :

x	35	36	37	38	39	40
f(x)	4.1	4.146	4.204	4.321	4.448	4.583

[7]

(c) Using Hermite's interpolation formula, find the value of $sin(0.5)$ from the following data :

x	sinx	cosx
-1	-0.8415	0.5403
0	0	1
1	0.8415	0.5403

[7]

4. (a) Evaluate the integral $\int_0^1 e^{-x^4} dx$ correct upto 4 decimal places by Simpson $\frac{1}{3}$ rule with step size $h = 0.2$. [7]

(b) Compute the value of integration $I = \int_0^1 \frac{1}{1+x^2} dx$ with the help of Romberg integration .Use only 3 initial values of integral with Trapezoidal rule. [7]

(c) Find a Cubic-spline curve to the following data set :

x:	-1	1	3	5
y:	2	4	11	23

Use end conditions $f''(-1) = 0$ and $f''(5) = 0$ [7]

5. (a) Solve the IVP $\frac{dy}{dx} = x^2 - sin(y)$,$y(0) = 1$ to compute $y(0.3)$ by Runge-Kutta method third order with step size 0.1. [7]

(b) Find the solution of first order IVP $\frac{dy}{dx} = x^2 + y$; $y(0) = 0$,for $x = 0.1$ with the help of Picard method .Compute only first four nonzero term. [7]

(c) Use Milne predicator method to compute $y(0.4)$ from the differential equation $\frac{dy}{dx} = y^2 - x^2$ and the foloowing values

x:	0	0.1	0.2	0.3
y:	1	1.11	1.25	1.42

[7]

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations June 2023

Programme: M.Sc. | **Session: 2022-23**

Semester: 4th | **Max. Time: 3 Hours**

Course Title: Number Theory | **Max. Marks: 70**

Course Code: SBSMAT 01 04 10 DCEC 3104

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (3.5X4=14)

a) Find the last two digits of the number 9^{9^9}.
b) Find the day of the week for October 4, 1995 and April 05, 2002.
c) For each positive integer n, show that $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$.
d) Express $\sqrt{73}$ in infinite continued fraction.
e) Find all the positive solution of $x^2 - 5y^2 = 1$ for $y < 100$.
f) Solve the congruence $x^2 \equiv -46(\text{mod } 17)$.
g) Solve the Legendre symbols $\left({}^{71}/_{73}\right)$ and $\left({}^{3658}/_{12703}\right)$.

Q 2. (2X7=14)

A. State and prove fundamental theorem of mathematics.
B. The quadratic congruence $x^2 + 1 = 0(\text{mod } p)$, where p is an odd prime, has a solution if and only if $p \equiv 1(\text{mod } 4)$.
C. State and prove Chinese Reminder Theorem.

Q 3. (2X7=14)

A. Define Euler's Phi-Function and prove it is multiplicative function. Calculate $\emptyset(5040)$.
B. For a positive integer n, derive the formula for $\tau(n)$ and $\sigma(n)$. Find $\tau(191)$ and $\sigma(210)$.

C. If n has a primitive root r and ind a denotes the index of a relative to r, then the following properties hold:

(i) $\operatorname{ind}(ab) \equiv \operatorname{ind} a + \operatorname{ind} b \pmod{\emptyset(n)}$.

(ii) $\operatorname{ind} a^k \equiv k \operatorname{ind} a \pmod{\emptyset(n)}$ for $k > 0$.

(iii) $\operatorname{ind} 1 \equiv 0 \pmod{\emptyset(n)}, \operatorname{ind} r \equiv 1 \pmod{\emptyset(n)}$.

Q 4. (2X7=14)

A. If p is an odd prime and a an odd integer, with $\gcd(a, p) = 1$, then prove that

$$({}^{a}/_{p}) = (-1)^{\sum_{k=1}^{p-1/2}[ka/p]}.$$

B. State and prove Quadratic Reciprocity Law.

C. Prove that any rational number can be written as a finite continued fraction. Express $[{}^{71}/_{55}]$ in continued fraction expansion.

Q 5. (2X7=14)

A. If $\frac{a}{b} < \frac{c}{d}$ are consecutive fractions in Farey sequence F_n, then prove that $bc - ad = 1$.

B. If ${}^{p_k}/_{q_k}$ are the convergent of the continued fraction expansion of $\sqrt{d}$ then prove that

$$p_k^2 - d\, d_k^2 = (-1)^{k+1} t_{k+1}, \text{ where } t_{k+1} > 0 \text{ for } k = 0, 1, 2, 3, \dots .$$

C. Show that any positive integer n can be written as the sum of four squares, some of which may be zero.

CENTRAL UNIVERSITY OF HARYANA

Second Semester Term End Examinations July 2023

Programme: **M.Sc. Mathematics** | **Session: 2022-23**
Semester: **2^{nd} Sem** | **Max. Time: 3 Hours**
Course Title: Numerical Analysis | **Max. Marks: 70**
Course Code: SBSMAT 01 02 03 C 3104

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.
2. Question no. 2 to 5 have three parts and student are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) The true value of e (exponential) correct to 10-significant digits is 2.718281828. Calculate absolute and relative errors, if we approximate this value by 2.718.

b) Define partial, scaled partial and complete pivoting for the solution of system of linear equations by Gauss elimination method.

c) Let $f(x)$ be a well defined real-valued function on the interval $[a, b]$ containing $(m+1)$ -points $x_0, x_1, \cdots, x_m$. If the function $f(x)$ is m-times differentiable in (a, b), then prove that there exists a point $\xi \in (a, b)$, such that

$$f[x_0, x_1, x_2, x_3, \ldots x_m] = \frac{f^m(\xi)}{m!}.$$

d) Compute the value of integration $I = \int_0^1 \frac{1}{1+x^2} dx$ with help of Romberg integration. Use 4 initial values 0.75, 0.775, 0.782794, 0.784747 of integral for Trapezoidal rule with *h=1, 0.5, 0.25, 0.125* respectively.

e) The growth of cell culture (optical density) at various pH levels are tabulated in the following table.

pH:	4	4.5	5	5.5	6
Optical density:	0.28	0.35	0.41	0.46	0.52

Compute the rate of change of optical density at pH level 4.

f) Obtain first three successive approximations of Picard method to the following IVP

$$\frac{dx}{dt} = t - x; \; x(0) = 1.$$

g) Compute the step size h for the IVP $\frac{dy}{dx} = -20y, \; y(0) = 1$, such that Euler method is stable.

Q 2.

(2X7=14)

a) Define order of convergence for a sequence, and compute it for Newton Raphson method.

b)

i) For what value of *K*, the iteration function $x_{n+1} = 2 - Kx_n + \left(\frac{K}{2} - 1\right)x_n^2$ will have quadratic convergence to the fixed point $\xi = 2$.

ii) Find the positive root of the equation $xe^x = 2$, which lies in the interval (0, 1) and correct to four decimal places. Use Secant method.

c) State the convergence condition for the Gauss-Seidel method, and interchange rows such that the following system satisfy convergence condition.

$$2x + 6y - z = 9$$
$$2x + y + 14z = -11$$
$$6x + 4y + z = 9$$

Then, perform 3 iterations of Gauss Seidel method to solve the resulting system starting with the initial approximation (0, 0, 0)

Q3.

(2X7=14)

a) The populations (in millions) of Punjab state up to two decimal points in the census years are given below. Estimate the populations for the years 1995 using Sterling and Bessel formulas.

Year	1971	1981	1991	2001	2011
Population	13.55	16.79	20.28	24.36	27.70

b) i) Using Rayleigh power method, determine the eigenvalue farthest to 4 for the matrix

$$A = \begin{bmatrix} 2 & 6 & -3 \\ 5 & 3 & -3 \\ 5 & -4 & 4 \end{bmatrix}$$

Start the iterations with the initial vector $X^{(0)} = [1 \quad 0 \quad 1]^T$. Perform only 2 iterations.

ii) Use the definitions of various finite differences to prove that $E^{1/2} \equiv \mu + \frac{\delta}{2}$.

c) Derive Hermite interpolation formula.

Q 4.

(2X7=14)

a) i) Estimate the integral $\int_0^{\frac{\pi}{2}} \sin(x)dx$ with Simpson 1/3 rules for number of subintervals $n = 6$ and find an upper bound of the error in estimation of the integral.

ii) Evaluate the integral $\int_1^2 (x^2 - \ln x)\, dx$ by using Gauss-Legendre 2-points formulas.

b) Use cubic spline fit to compute the value of $f(-0.5)$ for the following data points:

x:	-1	2	5
$f(x)$:	2	4	11

Use spline conditions $f''(-1) = 0$ and $f''(5) = 0$.

c) The tensile strengths of stainless steel cables of different diameters are investigated to give following results.

Cable diameter (x):	1.2	1.6	2.4	3.6	5.0	8.0
Tensile Strength(y):	1.32	2.53	5.28	8.34	12.83	23.92

Fit exponential curves to the data set.

Q 5. (2X7=14)

a) Solve the initial value problem $\frac{dr}{d\theta} + r^2 = \sin 2\theta;\ r(0) = 0$ by using 4th order Runge-Kutta method to compute the values of $r(0.2)$ with step size 0.2.

b) Solve the IVP $\frac{dy}{dx} = x^2 - \sin(y)$, to obtain the values of $y(0.4)$ and $y(0.5)$ using the Adams method. Given

$y(0.1) = 0.918507$ $\quad y(0.2) = 0.843754$ $\quad y(0.3) = 0.777677$

c) Solve the following BVP with the aid of finite difference method

$$y'' - 2xy' + 4y = 2x - 2; \qquad 0 \le x \le 0.6$$

$$y'(0) = 1, \qquad y(0.6) = -0.04.$$

Replace derivative boundary condition by central difference and divide the interval into three equal subintervals (or $h = 0.2$).

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations June 2023

Programme: UG **Session: 2022-23**

Semester: 2nd **Max. Time: 3 Hours**

Course Title: Vector Calculus **Max. Marks: 70**

Course Code: SBSMAT 03 02 01 GE 5106

Instructions:

1. Question no. 1 has seven parts and students are required to answer any five. Each part carries two marks.
2. Question no. 2 to 6 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (5X2=10)

a) Find a unit vector which is perpendicular to both (1, 0, 1) and (0, 1, 1).

b) Find the unit normal to the surface $xy^2 + 2yz = 4$ at the point (-2, 2, 3).

c) Find the gradient and Laplacian of $f = \sin(kx)\ \sin(ly)\ \exp(\sqrt{k^2 + l^2}z)$.

d) Write the vector equation $\boldsymbol{u} + (\boldsymbol{a} \cdot \boldsymbol{b})\boldsymbol{v} = |\boldsymbol{a}|^2(\boldsymbol{b} \cdot \boldsymbol{v})\boldsymbol{a}$ in suffix notation.

e) Define the Kronecker delta and the alternating tensor.

f) Define the Divergence theorem.

g) Show that $\varepsilon_{ijk}$is a tensor.

Q 2. (2X6=12)

a) Define dot product, cross product, scalar triple product and vector triple product with examples.

b) Define scalar and vector fields. Also, sketch the scalar field $T(x, y) = x^2 - y$

c) A particle with mass '*m*' and electric charge '*q*' moves in a uniform magnetic field '*B*'. Given that the force '*F*' on the particle is $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$, where '*v*' is the velocity of the particle moves at constant speed.

Q3. (2X6=12)

a) Define surface integral and then evaluate the surface integral of $\boldsymbol{u} = (xy, x, x + y)$ over the surface S defined by $z = 0$ with $0\ \leq x \leq 1, 0 \leq y \leq 2$, with the normal $\boldsymbol{n}$, directed in the positive z-direction.

b) Define line integral and evaluate the line integral over the curve 'C', i.e. $\int \boldsymbol{F}.\boldsymbol{dr}$, where $\boldsymbol{F}$=$(5z^2, 2x, x + 2y)$ and the curve 'C' is given by $x = t, y = t^2, z = t^2, 0 \leq t \leq 1$.

c) Show that both the divergence and the curl are linear operators i.e. $\nabla.(c\boldsymbol{u} + d\boldsymbol{v}) = c\nabla.\boldsymbol{u} + d\nabla.\boldsymbol{v}$ and $\nabla \times (c\boldsymbol{u} + d\boldsymbol{v}) = c\nabla \times \boldsymbol{u} + d\nabla \times \boldsymbol{v}$, where $\boldsymbol{u}$ and $\boldsymbol{v}$ are vector fields and c and d are constants.

Q 4. (2X6=12)

a) Using suffix notation, find an alternative expression (involving no cross products) for $\boldsymbol{a} \times \boldsymbol{b} \cdot \boldsymbol{c} \times \boldsymbol{d}$

b) Use divergence theorem to evaluate the surface integral $\iint_S \boldsymbol{v} \cdot \boldsymbol{n}\, \boldsymbol{ds}$ where $\boldsymbol{v} = (x + y,\ z^2, x^2)$ and S is the surface of hemisphere $x^2 + y^2 + z^2 = 1$ with $z > 0$ and $\boldsymbol{n}$ is the upward pointing normal. Note that the surface S is not closed.

c) State and prove Stokes's theorem with applications.

Q 5. (2X6=12)

a) Parabolic coordinates $(u,\ v,\ w)$ are related to Cartesian coordinates $(x_1,\ x_2,\ x_3)$ by the equations $x_1 = 2uv$, $x_2 = u^2\text{-}v^2$, $x_3 = w$. Sketch the u and v coordinate curves, find the scale factors h_u, h_v, h_w and the unit vectors e_u, e_v, e_w and check that $(u,\ v,\ w)$ coordinate system is orthogonal.

b) Find the formula for $\boldsymbol{\nabla f}$ in a general orthogonal curvilinear coordinate system by writing $\boldsymbol{\nabla f}$ in Cartesian coordinates and then finding the component of $\boldsymbol{\nabla f}$ in the $\boldsymbol{e_1}$ direction.

c) Find the divergence and curl of the unit vector $\boldsymbol{e_\phi}$ in spherical polar coordinates.

Q 6. (2X6=12)

a) If $\boldsymbol{\phi}$ is a scalar field, show that the quantity

$$T_{jk} = \frac{\partial^2 \phi}{\partial x_j \partial x_k}$$

is a second-rank tensor.

b) The most general isotropic second-rank tensor is a multiple of δ_{ij}.

c) The most general isotropic fourth-rank tensor is

$$a_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu \delta_{ik} \delta_{jl} + \nu \delta_{il} \delta_{jk}$$

Where λ, μ and ν are constants.

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations June 2023

Programme: UG **Session: 2022-23**

Semester: 2nd **Max. Time: 3 Hours**

Course Title: Vector Calculus **Max. Marks: 70**

Course Code: SBSMAT 03 02 01 GE 5106

Instructions:

1. Question no. 1 has seven parts and students are required to answer any five. Each part carries two marks.

2. Question no. 2 to 6 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (5X2=10)

a) Find a unit vector which is perpendicular to both (1, 0, 1) and (0, 1, 1).

b) Find the unit normal to the surface $xy^2 + 2yz = 4$ at the point (-2, 2, 3).

c) Find the gradient and Laplacian of $f = \sin(kx)\ \sin(ly)\ \exp\left(\sqrt{k^2 + l^2}z\right)$.

d) Write the vector equation $\boldsymbol{u} + (\boldsymbol{a} \cdot \boldsymbol{b})\boldsymbol{v} = |\boldsymbol{a}|^2(\boldsymbol{b} \cdot \boldsymbol{v})\boldsymbol{a}$ in suffix notation.

e) Define the Kronecker delta and the alternating tensor.

f) Define the Divergence theorem.

g) Show that $\varepsilon_{ijk}$is a tensor.

Q 2. (2X6=12)

a) Define dot product, cross product, scalar triple product and vector triple product with examples.

b) Define scalar and vector fields. Also, sketch the scalar field $T(x, y) = x^2 - y$

c) A particle with mass '***m***' and electric charge '***q***' moves in a uniform magnetic field '***B***'. Given that the force '***F***' on the particle is $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$, where '***v***' is the velocity of the particle moves at constant speed.

Q3. (2X6=12)

a) Define surface integral and then evaluate the surface integral of $\boldsymbol{u} = (xy, x, x + y)$ over the surface S defined by $z = 0$ with $0 \leq x \leq 1, 0 \leq y \leq 2$, with the normal ***n***, directed in the positive z-direction.

b) Define line integral and evaluate the line integral over the curve 'C', i.e. $\int \boldsymbol{F}.\boldsymbol{dr}$, where $\boldsymbol{F}$ =$(5z^2, 2x, x + 2y)$ and the curve 'C' is given by $x = t, y = t^2, z = t^2, 0 \leq t \leq 1$.

c) Show that both the divergence and the curl are linear operators i.e.

$\boldsymbol{\nabla}.(c\boldsymbol{u} + d\boldsymbol{v}) = c\boldsymbol{\nabla}.\boldsymbol{u} + d\boldsymbol{\nabla}.\boldsymbol{v}$ and $\boldsymbol{\nabla} \times (c\boldsymbol{u} + d\boldsymbol{v}) = c\boldsymbol{\nabla} \times \boldsymbol{u} + d\boldsymbol{\nabla} \times \boldsymbol{v}$, where ***u*** and ***v*** are vector fields and c and d are constants.

Q 4. (2X6=12)

a) Using suffix notation, find an alternative expression (involving no cross products) for $\boldsymbol{a} \times \boldsymbol{b} \cdot \boldsymbol{c} \times \boldsymbol{d}$

b) Use divergence theorem to evaluate the surface integral $\iint_S \boldsymbol{v} \cdot \boldsymbol{n}\, \boldsymbol{ds}$ where $\boldsymbol{v} = (x + y,\ z^2, x^2)$ and S is the surface of hemisphere $x^2 + y^2 + z^2 = 1$ with $z > 0$ and $\boldsymbol{n}$ is the upward pointing normal. Note that the surface S is not closed.

c) State and prove Stokes's theorem with applications.

Q 5. (2X6=12)

a) Parabolic coordinates (u, v, w) are related to Cartesian coordinates (x_1, x_2, x_3) by the equations $x_1 = 2uv$, $x_2 = u^2 - v^2$, $x_3 = w$. Sketch the u and v coordinate curves, find the scale factors h_u, h_v, h_w and the unit vectors e_u, e_v, e_w and check that (u, v, w) coordinate system is orthogonal.

b) Find the formula for $\boldsymbol{\nabla f}$ in a general orthogonal curvilinear coordinate system by writing $\boldsymbol{\nabla f}$ in Cartesian coordinates and then finding the component of $\boldsymbol{\nabla f}$ in the $\boldsymbol{e_1}$ direction.

c) Find the divergence and curl of the unit vector $\boldsymbol{e_\phi}$ in spherical polar coordinates.

Q 6. (2X6=12)

a) If $\boldsymbol{\phi}$ is a scalar field, show that the quantity

$$T_{jk} = \frac{\partial^2 \phi}{\partial x_j \partial x_k}$$

is a second-rank tensor.

b) The most general isotropic second-rank tensor is a multiple of δ_{ij}.

c) The most general isotropic fourth-rank tensor is

$$a_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu \delta_{ik} \delta_{jl} + \nu \delta_{il} \delta_{jk}$$

Where λ, μ and ν are constants.

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations June 2023

Programme: UG | **Session: 2022-23**

Semester: 2nd | **Max. Time: 3 hr**

Course Title: Multivariable Calculus | **Max. Marks: 70**

Course Code: SBSMAT 03 02 01 C 5106

Instructions:

1. Question no. 1 has seven parts and students are required to answer any five. Each part carries two Marks.
2. Question no. 2 to 6 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (5X2=10)

a) If $f(x,y) = \frac{xy^2}{x^2+y^4}$, does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist ?

b) If $\sin u = \frac{x^3+y^3}{x+y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$.

c) If $x = r\cos\theta\ \cos\phi, y = r\sin\theta\ \sin\phi,\ z = r\cos\theta$, then show that

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$$

d) Determine whether or not the vector field $\mathbf{F}(x,y,z) = \mathbf{i} + \sin z\ \mathbf{j} + y\cos z\ \mathbf{k}$ is conservative, if it is conservative then find a function 'f' such that $\mathbf{F} = \nabla f$.

e) Evaluate $\int_C \mathbf{F}.dr$ where $\mathbf{F}(x,y,z) = xy\,\mathbf{i} + yz\,\mathbf{j} + zx\,\mathbf{k}$ and C is the twisted cubic given by $x = t,\ y = t^2,\ z = t^3, \quad 0 \le t \le 1$.

f) If $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$ is a vector field on R^3, then find curl and divergence of **F**.

g) Define the Divergence theorem.

Q 2. (2X6=12)

a) The temperature at a point (x, y, z) is given

$$T(x,y,z) = 200\ e^{-x^2-3y^2-9z^2}$$

Where T is measured in °C and x, y, z in meters. Find the rate of change of temperature at the point P(2, -1, 2) in the direction toward the point (3, -3, 3). Also find the maximum rate of increase at P.

b) If $f(x,y) = e^x\ \cos y$, then find the directional derivative of 'f' at the point (0,0) in the direction indicated by the angle $\theta = \pi/4$.

c) If $f(x,y) = xyz^2 - 6$, then find the equation of tangent plane and normal line to the given surface at the point P(3, 2, 1).

Q3. (2X6=12)

a) Define Taylor's theorem and expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$.

b) Define Schwarz's and Young's theorem and show that for the function

$$f(x,y) = \begin{cases} \frac{x^2y^2}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$f_{xy}(0,0) = f_{yx}(0,0)$, even though the conditions of Schwarz's theorem and also of Young's theorem are not satisfied.

c) Prove that the function $f(x,y) = \sqrt{|xy|}$ is not differentiable at the point $(0,0)$ but f_x and f_y both exist at the origin and have the value '0'. Hence deduce that these two partial derivatives are continuous except at the origin.

Q 4. (2X6=12)

a) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point (3, 1, -1).

b) Find the extreme values of $f(x,y,z) = 3x - y - 3z$ subject to two constraints $x + y - z = 0$ and $x^2 + 2z^2 = 1$.

c) Define vector field and sketch the vector field **F** by drawing a diagram for the $\mathbf{F}(x,y) = \langle \frac{y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}} \rangle$.

Q 5. (2X6=12)

a) Evaluate $\int_C y\,dx + z\,dy + x\,dz$ where C consists of the segment C_1 from (2,0,0) to (3,4,5) followed by the vertical line segment C_2 from (3,4,5) to (3,4,0).

b) Suppose **F** is a vector field that is continuous on an open connected region D. If $\int_C \mathbf{F}.dr$ is independent of path in D, then **F** is a conservative vector field on D; that is there exists a function f such that $\nabla f = \mathbf{F}$.

c) State and prove Stokes' theorem.

Q 6. (2X6=12)

a) Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane and inside the cylinder $x^2 + y^2 = 2x$.

b) Use triple integration to find the volume of the tetrahedron T bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$ and $z = 0$.

c) Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2)\,dz\,dy\,dx$.

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, June-2023

Programme	: M.Sc. Mathematics	Session	: 2022-23
Semester	: II	Max. Time	: 3 Hours
Course Title	: Operations Research	Maximum Marks	: 70
Course Code	: SBSMAT 01 02 04 DCEC 3104		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each part carries three and half marks.
2. Question no. 2 to 5 have three parts and students need to answer any two parts of each question. Each part carries seven marks.

1. (a) Write short note on two phase method.
   (b) Prove that dual of the dual of a given primal, is the primal itself.
   (c) What is degeneracy in transportation problems? How is it resolved?
   (d) Explain Lowest cost entry method for finding initial basic feasible solution for a transportation problem.
   (e) Explain mean arrival rate, traffic intensity and average length of line.
   (f) State the assumptions made in sequencing problem.
   (g) Define saddle point, pure and mixed strategy.

2. (a) (i) Solve the following linear programming problem by graphical method.

$$\begin{aligned} Max.\ Z &= 4x_1 + 5x_2 \\ s.t. \quad x_1 + x_2 &\geq 1, \\ -2x_1 + x_2 &\leq 1, \\ 4x_1 - 2x_2 &\leq 1 \\ and \quad x_1, x_2 &\geq 0. \end{aligned}$$

   (ii) Discuss the scope and limitations of operations research.

   (b) Use dual simplex method to solve the following linear programming problem

$$\begin{aligned} Max. \quad Z &= -2x_1 - x_3 \\ s.t. \quad x_1 - 2x_2 + 4x_3 &\geq 8, \\ x_1 + x_2 - x_3 &\geq 5, \\ and \quad x_1, x_2, x_3 &\geq 0. \end{aligned}$$

   (c) The linear programming problem is

$$\begin{aligned} Max.\ Z &= 3x_1 + 5x_2 \\ s.t. \quad x_1 + x_2 &\leq 1, \\ 2x_1 + 3x_2 &\leq 1 \\ and \quad x_1, x_2 &\geq 0. \end{aligned}$$

   Find the variations in $c_j (j = 1, 2)$ which are permitted without changing the optimal solution.

3. (a) Explain Hungarian method for solving an assignment problem.
   (b) Determine the optimum basic feasible solution to the following transportation problem:

	To			Available
	6	8	4	14
From	4	9	3	12
	1	2	6	5
Required	6	10	15	

(c) Solve the travelling salesman problem in the matrix shown below:

	A	B	C	D	E	F
A	∞	20	23	27	29	34
B	21	∞	19	26	31	24
C	26	28	∞	15	36	26
D	25	16	25	∞	23	18
E	23	40	23	31	∞	10
F	27	18	12	35	16	∞

4. (a) For the $(M|M|1) : (FCFS, K)$ queuing model, show that the steady state probability, P_n is given by $P_n = \frac{\rho^n(1-\rho)}{(1-\rho^{K+1})}, 0 \leq n \leq K.$

(b) Obtain the steady state equations of the $(M/M/1) : (\infty/FCFS)$ queuing system. Also, find the probability that exactly n calling units are in the queuing system.

(c) Problems arrive at a computing center in Poisson fashion at an average rate of five per day. The rules of the computing centre are that man waiting to get his problem solved must aid the man whose problem is being solved. If the time to solve a problem with one man has an exponential distribution with mean time of 1/3 day, and if the average solving time is inversely proportional to the number of peoples working on the problem, approximate the expected time in the center of a person entering the line.

5. (a) State and prove minimax theorem for two-person zero-sum games. Solve the following game graphically.

		B: I	II	III	IV
	I	1	3	-3	7
A	II	2	5	4	-6

(b) Solve the game with the following payoff matrix by algebraically method.

	B		
	1	-1	-2
A	-1	1	1
	2	-1	0

(c) How will you solve the sequencing problem of n jobs on m machines. Solve the following sequencing problem.

	Machines		
Jobs	A	B	C
1	3	3	5
2	8	4	8
3	7	2	10
4	5	1	7
5	2	5	6

CENTRAL UNIVERSITY OF HARYANA

Second Semester Term End Examinations June-July 2023

Programme: Integ. BSc-MSc (Mathematics) **Session: 2022-23**

Semester: II **Max. Time: 3 Hour**

Course Title: Ordinary Differential Equations **Max. Marks: 70**

Course Code: SBSMAT 03 02 02 C 5106

Instructions:

1. *Question no. 1 has seven parts and students are required to answer any five. Each part carries two marks.*
2. *Question no. 2 to 6 have three parts and students are required to answer any two parts of each question. Each part carries six marks.*

Q 1(a). Solve the Bernoulli's equation: $\frac{dy}{dx} + y = xy^3$

Q 1(b). Find the singular solution of the differential equation: $y = 2xp - y p^2$

Q 1(c). Solve the equation: $(D-2)^2 y = 8\,(e^{2x} + \sin 2x + x^2)$

Q1 (d). Define Wronskian of n real functions $f_1, f_2, \ldots . f_n$. Prove the linear independence of fundamental set of solutions of $y''' - 2y'' - y' + 2y = 0$.

Q 1(e). Solve the homogeneous equation: $\frac{d^4x}{dt^4} + 4x = 0$

Q 1(f). A radioactive substance disintegrates at a rate proportional to the amount present at any instant. If M1 and M2 grams of the substance are present at times T1 and T2 respectively, find the expressions for decay constant (k), and half-life (T) of the substance.

Q 1(g). Define the ordinary, regular and irregular singular points of the differential equation.:

$$(x-1)\frac{d^2y}{dx^2} + (2x+3)\frac{dy}{dx} + 4xy = 0$$

Q 2(a). Test the exactness and solve: $(xy^3 + y)\,dx + 2\,(x^2 y^2 + x + y^4)\,dy = 0$

Q 2(b). Prove that the necessary and sufficient condition for the differential equation $M(x,y)dx + N(x,y)\,dy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Q 2(c). Using Picard's method of successive approximations to find the first three members of a sequence of functions that approaches the exact solution of the following IVP:

$$\frac{dy}{dx} = x^2 + y^2; \qquad y(0) = 1.$$

Q 3(a). Find the complete solution of $y'' - 2y' + 2y = x + e^x \cos x$

Q 3(b). Solve using method of Variation of Parameters: $(D^2 - 2D + 2)y = e^x \tan x$

Q 3(c). Given that y=x is a solution of $(x^2+1)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$, find a linearly independent solution by reducing the order.

Q 4(a). Solve the following equation: $(D+2)(D-1)^2 y = e^{-2x} + 2\sinh x$

Q 4(b). Solve using Method of Undetermined Coefficients: $(D^3 - 2D^2 + 4D)y = e^{2x} + \sin 2x$.

Q 4(c). Solve the Cauchy-Euler equation: $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$

Q 5(a). Find a power series solution about x=0 of the differential equation

$$xy'' + 2y' + xy = 0$$

Q 5(b). Using the generating function, prove that the Legendre's polynomial $P_n(x)$ satisfy the following recurrence relation: $n P_n(x) = xP'_n(x) - xP'_{n-1}(x)$

Q 5(c). Show that the Bessel's functions of first kind, $J_n(x)$, satisfy the following property: $[x^{-n} J_n(x)]' = -x^{-n} J_{n+1}(x)$, where ' denotes the differential.

Q6(a). Formulate the Lotka-Volterra predator-prey model, giving the relevant assumptions, model equations and interpretation of each term appearing in the equations. Find the equilibrium points for the system.

Q6(b). An uncharged condenser of capacity C is charged by applying an emf $E\sin(\frac{t}{\sqrt{LC}})$, through leads of self-inductance L and negligible resistance. Prove that at any time t, the charge on one of the plates is $\frac{EC}{2}\left\{\sin\left(\frac{t}{\sqrt{LC}}\right) - \frac{t}{\sqrt{LC}}\cos\left(\frac{t}{\sqrt{LC}}\right)\right\}$

Q6(c). Develop a very simple mathematical model which describes the exponential growth of a population. Write the model assumptions, governing differential equation and initial condition. Find the solution, and an expression for the time for the population to triple in size.

CENTRAL UNIVERSITY OF HARYANA

Second Semester Term End Examinations (Re-appear) June-July 2023

Programme: Integ. BSc-MSc (Mathematics) **Session: 2022-23**

Semester: II **Max. Time: 3 Hour**

Course Title: Differential Equations (P) **Max. Marks: 70**

Course Code: SBSMAT 03 02 02 C 4046

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.
2. Question no. 2 to 5 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1(a). Write down the UC sets corresponding to the functions: $(x^2 \sin 4x)$, $(e^{2x} \cos 3x)$.

Q 1(b). Test the exactness and solve: $(1 + 2xy \cos x^2 - 2xy)\, dx + (\sin x^2 - x^2)\, dy = 0$

Q 1(c). Find the integrating factor for the equation: $y\,(xy + 2x^2y^3)\, dx + x\,(xy - x^2y^2)\, dy = 0$

Q 1(d). Describe the Compartmental Model and the Balance Law.

Q 1(e). Solve the homogeneous equation: $\frac{d^4x}{dt^4} + 4x = 0$

Q 1(f). Find the Particular Integral of the equation: $(D + 2)(D - 1)^2 y = e^{-2x} + 2 \sinh x$

Q 1(g). Solve the Cauchy-Euler equation: $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3$

Q 2(a). Solve the Bernoulli's equation: $\frac{dx}{dy} - y\,x = y^3 x^2$

Q 2(b). Show that the differential equation $\mathbf{M}(x,y)dx + \mathbf{N}(x,y)\, dy = 0$ is exact iff $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Q 2(c). Solve the initial value problem: $(x^2 + 1)\frac{dy}{dx} + 4xy = x$; $y(2)=1$.

Q 3(a). Develop a very simple mathematical model which describes the exponential growth of a population. Write the model assumptions, governing differential equation and initial condition. Find the solution, and an expression for the time for the population to double in size.

Q 3(b). Solve the differential equation along with the initial condition for the concentration C of the pollutant in the lake, $\frac{dC}{dt} = \frac{F}{V}C_{in} - \frac{F}{V}C$, $C(0) = c_o$

How long will it take for the lake's pollution level to reach 5% of its initial level, if only fresh water flows into the lake.

Q 3(c). A certain radioactive salt decomposes at a rate proportional to the amount present at any instant. It has been found that 500 milligrams of the salt that was set aside 50 years ago has been reduced to 450 milligrams. How much of the salt will be left after 300 years?

Q 4(a). Solve the equation: $(D-2)^2 y = 8\,(e^{2x} + \sin 2x + x^2)$

Q 4(b). Find the complete solution of $\dfrac{d^4 y}{dx^4} + y = \tan x$

Q 4(c). Use the concept of Wronskian to prove the linear independence of fundamental set of solutions of $y''' - 2y'' - y' + 2y = 0$.

Q 5(a). Using method of Undetermined Coefficients, solve $\dfrac{d^2 y}{dx^2} - 2\dfrac{dy}{dx} - 3y = 2e^x - 10\sin x$.

Q 5(b). Solve the differential equation: $y'' - 2y' + 2y = x + e^x \cos x$

Q 5(c). Consider the Lotka-Volterra predator-prey model, assuming X and Y denote respectively the prey and predator population, $\dfrac{dX}{dt} = \beta_1 X - c_1 XY$, $\dfrac{dY}{dt} = -\alpha_1 X + c_2 XY$.

Here, c_1 *and* c_2 are interaction parameters, and β_1 is prey per-capita birth rate and α_2 is the predator per-capita death rate. Find the equilibrium points for the system. Draw the direction vector diagram for the phase-plane.

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, May/June-2023

Program	: M.Sc. Mathematics		
Semester	: Second	Session	: 2022-23
Course Title	: Typesetting in LaTeX	Max. Time	: 3 Hours
Course Code	: SBSMAT 01 02 05 C 2023	Maximum Marks	: 70

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. $(4 \times 3.5 = 14)$

(a) What is LaTeX. Discuss its advantages and disadvantages.

(b) $\mathcal{L}^{\Psi}_{\alpha,\Phi,\xi}(\pi;\eta') + \sum_{n=\kappa}^{\infty} \frac{n^{\kappa}+\Gamma^2}{\Gamma^{\Gamma}+\Upsilon}$.

(c) Discuss the command to draw a rectangle .Also specify the package used for it.

(d) Discuss the command to refer an equation with an example.

(e) Discuss the command to allign the equations in a document with an example.

(f) $\mathcal{B} = \{B_\alpha \in \mathcal{T} \mid U = \bigcup B_\alpha \forall U \in \mathcal{T}\}$

(g) Discuss the command to fill styles in pstrix with an example.

2. (a) Explain the LaTeX command for

$$\begin{Bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{Bmatrix} \times \begin{Vmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{Vmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mp} \end{bmatrix}$$

[7]

(b) Explain the LaTeX command for

$$\begin{aligned} g(x \mid \mu_i, \Sigma_i) &= \frac{1}{\sqrt{(2\pi)^D|\Sigma_i|}} \exp\Big(-\frac{1}{2}(x-\mu_i)' \\ &\qquad \times \Sigma_i^{-1}(x-\mu_i)\Big), \\ H_2 &= \sum_{\Omega} \log(|B(f_k, f_k)|) \end{aligned}$$

[7]

(c) Explain all the steps in detail to prepare and save an input file in LaTeXwith an example. [7]

3. (a) Discuss the cross references in articles with the help of examples in LaTeXalong with their outputs. [7]

(b) Supply the LaTeX command for table given below. [7]

DataSct	$\mathcal{J}$		$\mathcal{K}$		$\mathcal{L}$	
$\mathcal{D}_1$	2.1%	2.1%	2.1%	2.1%	2.1%	2.1%
$\mathcal{D}_2$	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%
$\mathcal{D}_3$	5.5%	5.5%	5.5%	5.5%	5.5%	5.5%

(c) How to prepare a bibliography and citation in LaTeX. Explain with the help of examples. [7]

4. (a) Design a document including article, ,titlepage,spaces,centering, graphics,multiline - aligned equations and flushing environment with the help of one example inLaTeX. [7]

(b) List all the packages which are used in LaTeXwith their specifications. [7]

(c) Discuss various types of shapes with the help of well known package.Also explain with the help of examples for different shapes. [7]

5. (a) Discuss Beamer class presentation including titlepage,outlines,list making environment,graphics, table and unnumbered equations. Also discuss one example. [7]

(b) What do you mean by Beamer in LaTeX.Discuss its advantages and disadvantages in detail. [7]

(c) Explain post script macros for generic tex(pstrix) with example. [7]

CENTRAL UNIVERSITY OF HARYANA

Second Semester Term End Examinations June 2023

Programme: M. Sc. (Mathematics) **Session: 2022-23**

Semester: Third **Max. Time: 3 Hours**

Course Title: Mathematical Statistics **Max. Marks: 70**

Course Code: SPMMAT 01 03 03 C 3104

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Questions no. 2 to 5 have three questions and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Explain with suitable examples the term "dispersion". State the relative and absolute measures of dispersion.

b) Show that if an event C is independent of two mutually exclusive events A and B, then C is also independent of $A \cup B$.

c) With usual notations, show that

$$Cov(aX + bY, cX + dY) = ac\,Var(X) + bd\,Var(Y) + (ad + bc)Cov(X, Y).$$

d) Show that the Geometric distribution has a memoryless property.

e) If X is a uniform random variable in [a, b], (a<b) then find the mean and variance of X.

f) State Central Limit Theorem.

g) Discuss in brief about null and alternative hypotheses.

Q 2. (2X7=14)

a) Calculate Interquartile Range, Quartile Deviation (Q.D.) and Coefficient of Quartile Deviation (Q.D.) from the following data:

Marks:	10	20	30	40	50	60
No. of Students	2	8	20	35	42	20

b) Find the coefficient of rank correlation from the following data:

X	15	10	20	28	12	10	16	18
Y	16	14	12	12	11	15	18	12

c) State and prove Baye's theorem.

Q3. (2X7=14)

a) Fit a Binomial distribution of the following data:

x:	0	1	2	3	4
f:	30	62	46	10	2

b) Derive expressions for the mean and variance of Poisson distribution.

c) Find the moment-generating function of the exponential distribution $f(x) = \frac{1}{c}e^{-\frac{x}{c}}, 0 \leq x < \infty, c > 0$. Hence find its mean and standard deviation.

Q 4. (2X7=14)

a) Let X be a random variable with the pdf $f(x) = \begin{cases} a\,e^{-\frac{x}{3}} & x > 0 \\ 0 & otherwise \end{cases}$. Find the (i) value of a (ii) $P(X > 3)$ (iii) $P(1 < X < 4)$.

b) Show that the mean deviation from the mean of the normal distribution is about 4/5 of its standard deviation.

c) Derive expressions for the mean and variance of Gamma Distribution.

Q 5. (2X7=14)

a) A random of 400 male students is found to have a mean height of 171.38 cm. Can it reasonably regarded as a sample from a large population with a mean height 171.17 cm and S.D. 3.30 cm? Use a 5% level of significance.

b) A sample analysis of the examination results of 500 students was made. It was found that 220 students had failed, 170 secured a third position, 90 secured a second division and the rest were placed in first position. Are these figures commensurate with the general examination result, which is in the ratio of 4:3:2:1 for various categories respectively? (The table value of chi-square for 3 d.f. at 5% l.o.s. is 7.815)

c) In a test given to two groups of students, the marks obtained are as follows:

First Group	18	20	36	50	49	36	34	49	41
Second Group	29	28	26	35	30	44	46		

Examine the significance of the difference between the mean marks secured by students of the above two groups. (The value of t at 5% level for 4 d.f. =2.14)

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations June-2023

Programme	: M.Sc. Mathematics (Re-appear)	Session	: 2022-23
Semester	: Fourth	Max. Time	: 3 Hours
Course Title	: Functional Analysis	Max. Marks	: 70
Course Code	: SBSMAT 01 04 09 DCEC 3104		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Define metric space and Gives its example.
   (b) Give an example of a Cauchy sequence which is not convergent in given space.
   (c) State and prove parallelogram law in Hilbert space.
   (d) Is every Hilbert spaces are Banach spaces?
   (e) An operator T on Hilbert space H is unitary if and only if it is an isometric isomorphism of H onto itself.
   (f) Define adjoint operator and gives its example.
   (g) An operator T on a Hilbert space H is normal operator if and only if $\|T^*(x)\| = \|T(x)\|$ for all $x \in H$.

2. (a) Prove that any contraction mapping f of a non-empty metric space (X, d) into itself has a unique fixed point.
   (b) Prove that every normed linear space is metric space and converse of this result is not true, gives an example .
   (c) Prove that L_p is a normed linear space where $1 \leq p < \infty$.

3. (a) Show that $C[0,1]$(set of all real continuous functions on [0,1]) is not reflexive space.
   (b) If M is a closed linear subspace of a normed linear space N and if $T : N \to N/M$ defined by $T(x) = x + M$. Show that T is continuous linear transformation for which $||T|| \leq 1$.
   (c) State and prove Cauchy-Schwarz Inequality.

4. (a) If M is a proper closed linear subspace of a Hilbert space H, then there exists a non zero vector $z_\circ$ in H such that $z_\circ \perp M$.
   (b) If A is a positive operators on a Hilbert space H then $(I + A)$ is non-singular where I is an identity operator.
   (c) Prove that the adjoint operator $T \to T^*$ on $\beta(H)$ has the following properties
   (i) $(T_1 + T_2)^* = T_1^* + T_2^*$ (ii) $(\alpha T)^* = \bar{\alpha} T^*$ (iii) $(T_1 T_2)^* = T_2^* T_1^*$.

5. (a) State and prove uniform boundedness principle.
   (b) Let N and N^1 be two normed linear space and D is a subspace of N. A linear transformation $T : D \to N^1$ is closed if and only if its Graph is closed.
   (c) Let B and $B_\circ$ be Banach spaces. If T is a continuous linear transformation of B onto $B_\circ$, then the image of each open sphere centred on the origin in B contains an open sphere centred on the origin, in $B_\circ$.

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Term End Examinations, June 2023

Programme: B.Sc.-M.Sc. (Mathematics) **Semester: 4th Semester**
Course Title: Object-Oriented Programming with C++ **Max. Time: 3 Hour**
Course Code: SBSMAT 03 04 01 SEC 3024 **Max. Marks: 70**

Instructions:

1. Question Number 1 (PART-I) is compulsory and carries total 10 marks (Each sub-part carries 2 Marks and attempt any five sub-parts).
2. Question Numbers 2 (two) to 6 (five) carry 12 marks each with internal choice.

Part-I

Q1:

a. What is the difference between C and C++ programming language?

b. What is the use of *new* operator in C++?

c. What do you mean by *variable*?

d. What do you mean by term *data abstraction*?

e. What is *dynamic polymorphism*?

f. List out the operators that *cannot be overloaded.*

g. How can class members be defined outside the class?

Part-II

Q 2.

(a) Explain the characteristics of object-oriented programming language? **(6)**

(b) What are the *different programming paradigm* in use? Explain the brief. **(6)**

(c) Write a C++ program to show the processing of One-dimensional Array using pointer. **(6)**

Q 3.

(a) What is *polymorphism* in OOP? Write a C++ program to overload the *sum* function **(6)**

(b) What are the benefits of using the *reference variable* in C++. Write a program to pass the class object to member function of class. **(6)**

(c) How can objects be *binding dynamically with pointer*? Explain with an example. **(6)**

Q 4.

(a) Explain the different types of the *constructor*. Write a C++ program using the *copy constructor*. **(6)**

(b) Define a class named 'BankAccount' to represent the following members:

Data members:-

- Account Number
- Name of Depositor
- Account Type

- Balance Amount

Member functions:

- Initialize members
- Deposit Amount
- Withdraw Amount
- Display Balance

Write a C++ program to test the Bank Account class for 2 customers. **(6)**

(c) How many types of *access specifiers* are available in C++. Differentiate them. **(6)**

Q 5.

(a) In C++, where is *Scope Resolution* (::) used? Give an example to illustrate the Scope Resolution Operator. **(6)**

(b) What are the benefits of using *static data members.* Give an example. **(6)**

(c) Write the *rules of operator overloading* as member function. **(6)**

Q 6.

(a) How will you *overload binary operator*? Write a C++ program to illustrate the *overloading binary operator.* **(6)**

(b) Write a C++ program to overload the ++ operator. **(6)**

(c) Write a C++ program to check whether *two objects are equal or not.* **(6)**